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ADDENDUM

Lie symmetries of nonlinear multidimensional reaction–diffusion systems: I

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Abstract. We provide some additional pairs of functions (F, G) that do not appear in our original paper.

Recently, the authors have found that in tables 1 and 5 of their paper [1] some pairs of functions (F, G) were missed which lead to extensions of the trivial Lie algebra $AE(1, n)$ for *nonlinear* systems of the form

$$\begin{aligned} \lambda_1 U_t &= \Delta U + F(U, V) \\ \lambda_2 V_t &= \Delta V + G(U, V). \end{aligned} \tag{1}$$

(Here F and G are arbitrary smooth functions, $U = U(t, x)$, $V = V(t, x)$ are unknown functions of $n + 1$ variables t, $x = (x_1, \dots, x_n)$, Δ is the Laplacian, and the t subscript to the functions U and V denotes differentiation with respect to this variable.) To achieve completeness of tables 1–5 of [1], the missing pairs of the functions (F, G) are listed in an additional table 6 below.

Remark. All notations used in table 6 coincide with that in tables 1–5, while the functions $y_k(t)$, $k = 1, 2$, form a fundamental system of solutions of the linear second-order equation

$$\lambda_1 \lambda_2 \frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} - (\lambda_2 \beta_1 + \lambda_1 \beta_{20}) \frac{\mathrm{d}y(t)}{\mathrm{d}t} + (\beta_1 \beta_{20} - \beta_0 \beta_{10}) y(t) = 0. \tag{2}$$

The appropriate forms for the $y_k(t)$ depend on the coefficients of equation (2).

It should be noted that cases 2, 3 and 4 (see table 6) are natural continuations of cases 1, 2 and 3 of table 1, respectively, while cases 5, 6, and 19 represent quite new systems with the nonlinearity UV. On the other hand, cases 7 and 9 are special subcases of the system 9 (see table 5); similarly cases 10–12 and 20 are obtained from the systems 6 and 7 (see table 5) with $\beta_{20} = 0$. Other cases listed in table 6 are also special subcases of the relevant systems from the original paper.

Finally, we note that the coefficient α_1 can vanish in the systems 3 and 9 (see table 3); analogously one can have $\beta_2 = 0$ in system 4 (see table 2).

§ Note that Dr Cherniha's e-mail address was printed incorrectly in the original paper [1].

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Table 6. Nonlinear systems of the form (1) with non-trivial Lie algebras ($\lambda_1 \neq 0$) .

	Nonlinearities	Restrictions	Basic operators of MAI
1	$F = \beta_1 U + \beta_{10} V^{\alpha}$ $G = \beta_2 V$	$\beta_2 \neq 0$ $\alpha \neq 0; 1; \lambda_1/\lambda_2$	$AE(1.n)$ $X^{\infty}_{\beta_1} = P_{\beta_1}(t, x)\partial_U,\ Q_{\alpha} = \alpha U\partial_U + V\partial_V$
2	$F = \beta_1 U + \beta_{10} V^{\lambda_1/\lambda_2}$ $G = \beta_2 V$	$\lambda_2\beta_1\beta_2 \neq 0$ $\beta_1\lambda_2^2 \neq \beta_2\lambda_1^2$	$AE(1.n),\ X^{\infty}_{\beta_1}$ $Q_{\lambda} = \lambda_1 U\partial_U + \lambda_2 V\partial_V,\ G_a = tP_a - \frac{x_a}{2} Q_{\lambda}$
3	$F = \beta_{10} V^{\lambda_1/\lambda_2}$ $G = 0$	$\lambda_2 \neq 0$	$AE(1.n),\ X^{\infty}_1 = P_1(t, x)\partial_U$ $Q_{\lambda},\ G_a,\ D = 2tP_t + x_a P_a - \frac{2\lambda_2}{\lambda_1} V\partial_V$
4	$F = \beta_{10} V^{1+4/n}$ $G = 0$	$\frac{\lambda_1}{\lambda_2} = 1 + \frac{4}{n}$	$AE(1.n),\ X^{\infty}_1$ $Q_{\lambda},\ G_a,\ D = 2tP_t + x_a P_a - I_n,\ \Pi$
5	$F = \beta_0 UV + \beta_1 U$ $+ \beta_{10} U \log U$ $G = 0$	$\lambda_2 = 0$ $\beta_0\beta_{10} \neq 0$	$AE(1.n)$ $Q^{\infty}_{\beta} = T(t)(U\partial_U - \frac{\beta_{10}}{\beta_0}\partial_V) + \frac{\lambda_1}{\beta_0} T_t(t)\partial_V$ $G^{\infty}_a = (\int T(t)\,\mathrm{d}t)\partial_a - \frac{x_a}{2} Q^{\infty}_{\beta}$
6	$F = \beta_0 UV$ $G = 0$	$\lambda_2 = 0$	$AE(1.n),\ Q^{\infty}_{\beta}, G^{\infty}_a$ at $\beta_{10} = 0$ $D_- = 2tP_t + x_a P_a - 2V\partial_V$ $\Pi_{\beta} = tD - t^2\partial_t - \frac{\lambda_1\|x\|^2}{4} U\partial_U + \frac{\lambda_1 n}{2\beta_0}\partial_V$
7	$F = \beta_1(\alpha U - V)^{1-\alpha_0}$ $G = \beta_2(\alpha U - V)^{1-\alpha_0}$	$\alpha_0\alpha \neq 0$ $\alpha_0 \neq 1$	$AE(1.n),\ Q^{\infty}_{\alpha} = R_0(x)(\partial_U + \alpha\partial_V)$ $D_1 = 2tP_t + x_a P_a + \frac{2}{\alpha_0}(U\partial_U + V\partial_V)$
8	$F = \beta_1 U^{1-\alpha_0}$ $G = \beta_2 V U^{-\alpha_0}$	$\lambda_2 = 0$ $\alpha_0\beta_2 \neq 0$	$AE(1.n),\ D_1,\ I^{\infty} = T(t)V\partial_V$
9	$F = \beta_1 \log(\alpha U - V) + c_1$ $G = \beta_2 \log(\alpha U - V) + c_2$	$\alpha \neq 0$	$AE(1.n),\ Q^{\infty}_{\alpha},\ D_{\mathrm{esp}} = 2tP_t + x_a P_a$ $+2(U\partial_U + V\partial_V) + \Big[2t\frac{\beta_2-\alpha\beta_1}{\alpha(\lambda_2-\lambda_1)}$ $+\|x\|^2\frac{\beta_2\lambda_1-\alpha\beta_1\lambda_2}{n\alpha(\lambda_2-\lambda_1)}\Big](\partial_U + \alpha\partial_V)$
10.	$F = \beta_1 \log V$ $G = c_2$	$c_2 \neq 0$	$AE(1.n),\ X^{\infty}_1,\ D_{\beta} = 2tP_t + x_a P_a$ $+2(U\partial_U + V\partial_V) + \frac{2\beta_1 t}{\lambda_1}\partial_U$
11.	$F = \beta_1 \log V$ $G = 0$	$\lambda_2 \neq 0$	$AE(1.n),\ X^{\infty}_1$ $D_{\beta},\ Y_{\beta} = \lambda_1 V\partial_V + \beta_1 t\partial_U$
12.	$F = \beta_1 \log V$ $G = 0$	$\lambda_2 = 0$	$AE(1.n),\ X^{\infty}_1$ $D_{\beta},\ Y^{\infty}_{\beta} = \lambda_1 T(t)V\partial_V + \beta_1(\int T(t)\,\mathrm{d}t)\partial_U$
13	$F = 0$ $G = \beta_0 U + \beta_{20} U^{\alpha}$	 $\beta_0 \neq 0$	$AE(1.n),\ X^{\infty}_2 = P_2(t, x)\partial_V$ $D_+ = 2tP_t + x_a P_a + 2V\partial_V$
14	$F = \beta_1 \exp U$ $G = \beta_2 V \exp U$	$\lambda_2 = 0$ $\beta_2 \neq 0$	$AE(1.n),\ I^{\infty}$ $D_2 = 2tP_t + x_a P_a - 2\partial_U$
15	$F = \beta_1 \exp V$ $G = \beta_2$	$\lambda_2 = 0$ $\beta_2 \neq 0$	$AE(1.n),\ X^{\infty}_1,\ Q_1 = \lambda_1(U\partial_U + \partial_V)$ $G_{1a} = tP_a - \frac{x_a}{2} Q_1$
16	$F = \beta_1 \exp V$ $G = 0$	$\lambda_2 = 0$	$AE(1.n),\ Q_1,\ X^{\infty}_1$ $G_{1a},\ D_3 = 2tP_t + x_a P_a - 2\partial_V$
17	$F = \beta_1$ $G = \beta_2 \exp U$	$\lambda_2\beta_1 \neq 0$	$AE(1.n),\ X^{\infty}_2$ $Q = \partial_U + V\partial_V$

Table 6. (Continued)

	Nonlinearities	Restrictions	Basic operators of MAI
18	$F = \beta_1$ $G = \beta_2 \exp U$	$\lambda_2 = 0$	$AE(1.n),\ X_2^\infty$ $Q^\infty = T(t)(\partial_U + V\partial_V)$
19	$F = \beta_1 U + \beta_{10} \log V$ $G = \beta_0 UV + \beta_2 V$ $+ \beta_{20} V \log V$	$\lambda_2\beta_0 \neq 0$ β_{10} or $\beta_{20} \neq 0$	$AE(1.n),\ Y_k = y_k(t)(\beta_0 V\partial_V$ $-\beta_{20}\partial_U) + \frac{\mathrm{d}y_k(t)}{\mathrm{d}t}\partial_U, k = 1, 2$
20	$F = \beta_1 U + \beta_{10} \log V$ $G = \beta_2 V$	$\lambda_2 = 0$ β_2 or $\beta_1 \neq 0$	$AE(1.n),\ X_{\beta_1}^\infty$ $Y_{\beta_1}^\infty = \lambda_1 T(t) V\partial_V$ $+\beta_{10} \exp \frac{\beta_1 t}{\lambda_1}\left[\int T(t) \exp(-\frac{\beta_1 t}{\lambda_1})\mathrm{d}t\right]\partial_U$

Reference

[1] Cherniha R and King J R 2000 Lie symmetries of nonlinear multidimensional reaction–diffusion systems: I *J. Phys. A: Math. Gen.* **33** 267–82